

Beyond Algorithms: A Portrait of Elementary Teachers' Pedagogical Content Knowledge in Fractions

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Abstract

This study explores elementary teachers' Pedagogical Content Knowledge in the domain of fractions by analyzing their responses to fraction-based numeracy tasks and semi-structured interviews. Using a qualitative descriptive approach, five elementary teachers were asked to solve and explain fraction problems involving comparison, equivalence, addition, student misconceptions, and curriculum connections. The analysis focused on four dimensions of PCK: Content Knowledge, Knowledge of Students, Knowledge of Instructional Strategies, and Knowledge of Curriculum. The findings reveal that although all participants demonstrated procedural fluency in solving fraction tasks, their pedagogical orientations varied considerably. Two teachers exhibited a Procedural-Focused orientation, relying primarily on algorithmic procedures with limited conceptual explanation or use of representations. One teacher demonstrated an Algorithmic-Partial orientation, characterized by heavy dependence on rules and the presence of misconceptions. Two teachers showed more advanced orientations: one displayed Conceptual-Reflective PCK through the use of visual representations and anticipation of student misconceptions, while another demonstrated Integrated PCK by balancing procedural accuracy with conceptual understanding, effective representations, and explicit alignment with the national curriculum. These findings indicate that algorithmic competence alone is insufficient to support meaningful learning of fractions. Teachers who integrated conceptual explanations, representations, and curricular awareness were better positioned to address student thinking and misconceptions. This study contributes to the literature by illustrating a spectrum of teachers' PCK profiles in the Indonesian context and highlights the need for professional development programs that support teachers in moving beyond algorithm-focused instruction toward more integrated and conceptually grounded approaches to teaching fractions.

Keywords:

Pedagogical Content Knowledge,
Teachers,
Fractions,
Numeracy,
Elementary education

1. Introduction

Fractions are widely recognized as one of the most conceptually demanding topics in elementary mathematics and play a central role in the development of numeracy. Unlike whole numbers, understanding fractions requires students to reason about part whole relationships, ratios, and proportionality (Lamon, 2007). Research has consistently shown that students often experience difficulties in transitioning from whole-number reasoning to fractional reasoning, leading to common misconceptions such as treating the numerator and denominator as independent whole numbers or assuming that a larger denominator indicates a larger fraction (Behr et al., 1983). These persistent misconceptions can negatively affect students' performance in proportional reasoning, algebra, and

more advanced mathematical learning, positioning fractions as a “gatekeeper” concept in mathematics education (Siegler et al., 2011). Consequently, improving the teaching and learning of fractions remains an important focus of mathematics education research worldwide.

Teachers play a critical role in supporting students’ conceptual understanding of fractions. However, numerous studies have reported that many elementary teachers encounter difficulties in teaching fractions conceptually. Ma (1999) found that even experienced teachers often rely on memorized rules and procedures rather than deep conceptual understanding, particularly in fraction topics. Similarly, Charalambous and Pitta-Pantazi (2007) reported that instruction on fractions is frequently dominated by algorithmic procedures, which may lead students to perform operations mechanically without meaningful understanding. Although procedural fluency is an essential component of mathematical competence, an overemphasis on algorithms can limit students’ ability to reason flexibly and apply fraction knowledge across contexts. These findings highlight the need to examine teachers’ knowledge beyond content mastery alone.

Shulman’s (1986) concept of Pedagogical Content Knowledge provides a comprehensive framework for understanding the specialized knowledge teachers require to make subject matter accessible to learners. PCK encompasses several interrelated dimensions, including Content Knowledge, Knowledge of Students, Knowledge of Instructional Strategies, and Knowledge of Curriculum. In the context of fractions, effective PCK involves not only understanding fraction concepts but also anticipating students’ misconceptions, selecting appropriate representations, and aligning instructional strategies with curricular expectations. Subsequent work on Mathematical Knowledge for Teaching by Ball, Thames, and Phelps (2008) and Hill, Rowan, and Ball (2005) further emphasizes that teachers’ specialized mathematical knowledge is closely associated with students’ learning outcomes. Together, these perspectives offer a robust theoretical foundation for analyzing the complexity of fraction instruction.

Despite the growing body of research on fractions and PCK, several important gaps remain. Much of the existing literature has been conducted in Western educational contexts, providing limited empirical evidence from settings such as Indonesia, where national curricula emphasize numeracy and real-world applications but classroom instruction often remains procedural. In addition, prior studies tend to examine teachers’ knowledge or instructional practices in isolation, with relatively few studies adopting an integrated perspective to map teachers’ holistic PCK in teaching fractions. Moreover, qualitative research that captures the diversity of teachers’ PCK orientations from procedural-focused to integrated conceptual approaches based on authentic tasks and reflective reasoning remains limited. Addressing these gaps is essential for informing the design of professional development programs that support teachers in balancing procedural fluency with conceptual understanding.

This study aims to contribute to this line of inquiry by examining elementary teachers’ Pedagogical Content Knowledge in teaching fractions through the analysis of their responses to fraction-based numeracy tasks and semi-structured interviews. By identifying and categorizing teachers’ PCK profiles across the dimensions of CK, KoS, KoIS, and KoC, the study seeks to illuminate how these forms of knowledge are integrated in instructional decision-making. Focusing on the Indonesian context, this research provides empirical evidence that extends existing theoretical discussions on PCK and offers practical insights for the development of teacher education and professional learning programs oriented toward conceptual and reflective fraction instruction.

2. Method

This study employed a qualitative descriptive design to explore how elementary teachers demonstrated their Pedagogical Content Knowledge in teaching fractions. A qualitative approach was chosen because it enables an in-depth examination of teachers’ reasoning, representations, and pedagogical decisions, which cannot be adequately captured through quantitative methods alone (Creswell & Poth, 2018). The study was conducted within a graduate program in elementary education at a state university in Indonesia. Participants consisted of five elementary school teachers who were enrolled in a master’s program in primary education and had prior experience teaching in elementary classrooms. Their dual position as practicing teachers and graduate students provided a relevant context for examining how PCK is enacted in fraction-related instructional tasks.

Data were collected over a two-month period during the 2025 academic year using two main instruments: fraction-based numeracy tasks and semi-structured interviews. The numeracy tasks were

designed to elicit teachers' mathematical reasoning, problem-solving strategies, use of representations, and curricular awareness in the context of fractions. The tasks focused on key fraction concepts commonly taught in elementary school, including comparison, equivalence, and operations.

The fraction numeracy test consisted of five items, each targeting specific dimensions of PCK:

- (1) Fraction comparison (e.g., comparing $\frac{3}{8}$ and $\frac{5}{8}$) to assess Content Knowledge and Knowledge of Instructional Strategies;
- (2) Fraction equivalence (e.g., explaining why $\frac{2}{3} = \frac{4}{6}$ to a Grade 5 student) to assess CK and Knowledge of Students;
- (3) Fraction operations (e.g., solving $\frac{2}{3} + \frac{3}{4}$ and explaining the procedure) to assess CK and KoIS;
- (4) Student misconception scenario (e.g., responding to the claim that $\frac{1}{6} > \frac{1}{4}$ because 6 is larger than 4) to assess KoS and KoIS; and
- (5) Curriculum connection, which asked participants to identify grade-level placement of fraction concepts and instructional sequencing in the Indonesian curriculum, to assess Knowledge of Curriculum.

Following completion of the written tasks, semi-structured interviews were conducted to further probe participants' justifications, instructional choices, and reflections on how they would teach the tasks to their students. Interview prompts included questions such as "Why did you choose this strategy?", "What difficulties might students experience?", and "How does this task align with the curriculum?". Each interview lasted between 45 and 60 minutes and was audio-recorded with participants' consent.

Instrument validity was established through expert judgment by two mathematics education lecturers, who reviewed the tasks and interview questions for content relevance and clarity. Data analysis followed a thematic coding procedure (Braun & Clarke, 2006). Participants' written responses and interview transcripts were transcribed verbatim and coded according to four PCK dimensions: Content Knowledge, Knowledge of Students, Knowledge of Instructional Strategies, and Knowledge of Curriculum. Based on recurring patterns across these dimensions, participants were categorized into distinct PCK profiles, such as Procedural-Focused, Conceptual-Reflective, and Integrated PCK. To enhance trustworthiness, data triangulation was conducted by comparing task responses and interview data, and peer debriefing was carried out with a senior mathematics education researcher.

Overall, this methodological approach enabled a nuanced analysis of teachers' PCK in fractions by moving beyond surface-level procedural performance to uncover underlying reasoning and pedagogical orientations.

3. Results and Discussion

3.1 Results

The analysis of teachers' responses to the five fraction tasks revealed substantial variation in their Pedagogical Content Knowledge. Each task was designed to elicit specific dimensions of PCK, including Content Knowledge, Knowledge of Students, Knowledge of Instructional Strategies, and Knowledge of Curriculum. The results are presented according to the tasks administered.

Task 1: Fraction Comparison $\frac{3}{4}$ and $\frac{5}{8}$

All participants correctly identified that $\frac{3}{4}$ is greater than $\frac{5}{8}$; however, the strategies used differed markedly. P1 and P2 relied on procedural methods by finding a common denominator (8) and converting $\frac{3}{4}$ to $\frac{6}{8}$, without employing visual representations. In contrast, P3 and P5 used area models and number lines to justify their comparisons, explaining that $\frac{3}{4}$ represents a larger portion of the whole than $\frac{5}{8}$. P4 applied cross-multiplication to reach the correct answer but did not provide a conceptual explanation.

Task 2: Fraction Equivalence $\frac{2}{3} = \frac{4}{6}$

Participants demonstrated varying levels of understanding regarding fraction equivalence. P1 and P2 justified equivalence through procedural transformation by multiplying the numerator and denominator by two, but they did not explain why the procedure preserves value. P3 and P5 used visual representations, such as partitioning shapes into equal parts, to show that both fractions represent the same quantity. P4 correctly performed the calculation but reported difficulty in explaining the concept to students. This task revealed the need for teachers to link procedural transformations with conceptual meaning, a hallmark of Emerging Conceptual or Integrated PCK.

Task 3: Fraction Addition $\frac{2}{3} + \frac{3}{4}$

Differences in procedural fluency and conceptual justification were evident in this task. P1 and P2 applied the common denominator method and obtained the correct result (17/12) but did not provide conceptual explanations. P3 combined algorithmic steps with an area model to explain the meaning of addition as combining parts of a whole. P4 initially produced an incorrect answer (5/7), reflecting a misconception in adding numerators and denominators directly. P5 solved the problem correctly and explained the result both procedurally and conceptually, including its conversion to a mixed number (1 5/12).

Task 4: Student Misconception ($\frac{1}{6} > \frac{1}{4}$ because $6 > 4$)

When responding to the misconception that 1/6 is greater than 1/4 due to the larger denominator, P1 and P2 provided corrective explanations verbally but did not use visual representations. P3 and P5 addressed the misconception using area models, illustrating that dividing a whole into fewer parts results in larger portions. They also identified whole-number reasoning as a common source of student error. P4 expressed uncertainty and offered a brief explanation without elaboration.

Task 5: Curriculum Connection

Responses to the curriculum connection task indicated uneven Knowledge of Curriculum among participants. P1 and P4 were uncertain about the grade levels at which fraction concepts are introduced. P2 identified Grade 4 but provided limited justification. In contrast, P3 and P5 accurately described the placement of fraction comparison and equivalence in Grade 4 and fraction operations with unlike denominators in Grades 5–6, emphasizing instructional sequencing from concrete representations to abstract procedures.

Tabel 1. Summary Table of PCK Profiles

Participant	Dominant Orientation	Key Features
P1	Procedural-Focused	Correct procedures; minimal use of representations; limited KoC
P2	Procedural-Focused	Accurate algorithmic work; weak in conceptual justification
P3	Conceptual-Reflective	Effective use of models; anticipates misconceptions; links to curriculum
P4	Algorithmic-Partial	Reliance on procedures; occasional errors; limited student and curriculum insight
P5	Integrated PCK	Balances procedures and concepts; uses representations; strong curricular links

Overall, the results illustrate a spectrum of PCK orientations among the participants. While most teachers demonstrated procedural competence in fraction tasks, fewer integrated conceptual explanations, representations, and curricular considerations consistently. Only one participant exhibited a fully integrated PCK profile across all tasks.

3.2. Discussion

The findings of this study reaffirm that fractions constitute a persistent challenge not only for students but also for elementary teachers, as evidenced by the diversity of Pedagogical Content Knowledge profiles identified among participants. Although all teachers demonstrated procedural competence in fraction tasks, only a limited number were able to consistently integrate conceptual explanations, representations, and curricular awareness. This pattern strongly supports Shulman's (1986) foundational claim that effective teaching depends on the integration of multiple forms of professional knowledge, rather than on content mastery or procedural fluency alone.

Across the data, a clear distinction emerged between teachers whose instruction was dominated by algorithmic procedures and those who employed conceptually grounded and representational approaches. Teachers categorized as Procedural-Focused or Algorithmic-Partial tended to rely on symbolic manipulation and rules, often without articulating underlying meanings or pedagogical rationales. Prior research has shown that such instruction can contribute to students' fragmented understanding of fractions, where operations are performed mechanically without reference to quantities

or relationships (Behr et al., 1983; Lamon, 2007). In contrast, teachers demonstrating Conceptual-Reflective and Integrated PCK consistently used visual models, such as area representations and number lines, to support students' part whole reasoning. This finding aligns with Ball, Thames, and Phelps' (2008) conceptualization of Mathematical Knowledge for Teaching, which emphasizes teachers' ability to select, justify, and connect representations as a key component of effective mathematics instruction.

The role of Knowledge of Students emerged as particularly salient in the domain of fractions. Teachers with stronger PCK were able to anticipate and explicitly address common misconceptions, especially those arising from whole-number bias, such as the belief that a larger denominator implies a larger fraction. Research has consistently identified whole-number reasoning as a major cognitive obstacle in the learning of rational numbers (Ni & Zhou, 2005; Siegler et al., 2011). In this study, only teachers with Conceptual-Reflective and Integrated PCK demonstrated the pedagogical capacity to diagnose these misconceptions and respond using representations that made relative magnitudes visible. As Hill, Rowan, and Ball (2005) argue, the ability to interpret and respond to student thinking is a defining characteristic of high-quality PCK and is closely linked to student learning outcomes.

In addition to KoS, variation in Knowledge of Instructional Strategies further differentiated teachers' PCK profiles. Teachers with limited PCK tended to treat instructional strategies as procedural demonstrations, whereas those with more integrated knowledge approached instruction as a process of meaning-making supported by multiple representations and explanations. This distinction echoes Ma's (1999) observation that teachers with profound understanding of fundamental mathematics are better positioned to flexibly adapt instruction and provide coherent explanations. The present findings suggest that the absence of such flexibility may constrain teachers' ability to move beyond rule-based teaching, particularly in conceptually demanding topics such as fraction equivalence and operations.

Knowledge of Curriculum also played a critical role in shaping teachers' instructional orientations. Only a subset of participants demonstrated accurate awareness of the sequencing of fraction concepts within the Indonesian curriculum, including the progression from comparison and equivalence to operations with unlike denominators. Teachers lacking this curricular perspective risk introducing algorithms prematurely or without sufficient conceptual grounding, a practice that has been shown to undermine students' long-term understanding of rational numbers (Lamon, 2007; Siegler et al., 2011). Consistent with Shulman's (1986) framework, these findings underscore that curricular knowledge is not a peripheral component of PCK but an essential element that guides instructional decision-making and coherence.

Taken together, the identified PCK profiles Procedural-Focused, Algorithmic-Partial, Conceptual-Reflective, and Integrated PCK, illustrate a continuum of instructional orientations among elementary teachers. This continuum reflects broader patterns documented in international research, where teachers often begin with procedural approaches and gradually develop more integrated forms of knowledge through experience and professional learning (Ball et al., 2008; Charalambous & Pitta-Pantazi, 2007). While procedural fluency remains a necessary foundation, the findings clearly indicate that it is insufficient for fostering deep, transferable understanding of fractions. Integrated PCK, characterized by the coordination of content knowledge, student thinking, instructional strategies, and curriculum, emerged as the most robust orientation for supporting meaningful fraction learning.

These findings carry important implications for teacher professional development. Rather than emphasizing procedural mastery alone, professional learning programs should be designed to explicitly engage teachers in analyzing student misconceptions, working with multiple representations, and reflecting on curricular progression. Research suggests that professional development models such as lesson study, task-based workshops, and structured analysis of student work can effectively support the development of PCK by situating teacher learning in authentic instructional contexts (Lewis, Perry, & Murata, 2006; Darling-Hammond et al., 2017). By engaging teachers in reflective examination of their instructional decisions, such approaches may facilitate the transition from procedural-focused practices toward more integrated and conceptually oriented forms of PCK.

In this sense, advancing fraction instruction does not require abandoning algorithms, but rather repositioning them within richer pedagogical repertoires that emphasize meaning, representation, and coherence. By situating procedural fluency within a broader conceptual and curricular framework, teachers can better support students in understanding, applying, and reasoning with fractions in ways that are both mathematically sound and educationally meaningful.

4. Conclusion

This study examined elementary teachers' Pedagogical Content Knowledge in the domain of fractions through an analysis of their responses to fraction-based numeracy tasks and semi-structured interviews. The findings indicate that although all participants demonstrated procedural fluency, their orientations toward teaching fractions differed substantially. Two participants (P1 and P2) exhibited a Procedural-Focused orientation, relying predominantly on algorithmic procedures with limited conceptual explanation or use of representations. One participant (P4) demonstrated an Algorithmic-Partial orientation, characterized by dependence on rules alongside occasional conceptual misconceptions. In contrast, two participants (P3 and P5) displayed more advanced orientations. P3 demonstrated Conceptual-Reflective PCK by integrating visual representations and anticipating student misconceptions, while P5 achieved Integrated PCK by balancing procedural accuracy with conceptual clarity, effective use of representations, and explicit curricular alignment. These profiles illustrate the spectrum of PCK in practice and reinforce the importance of moving beyond algorithmic instruction toward a more integrated coordination of Content Knowledge, Knowledge of Students, Knowledge of Instructional Strategies, and Knowledge of Curriculum.

The findings of this study offer important implications for mathematics education, particularly in the design of teacher professional development. The results underscore the need for professional learning programs that extend beyond procedural competence to emphasize conceptual understanding, strategic use of representations, and the ability to anticipate and address common student misconceptions in fractions. Consistent with prior research on effective mathematics instruction, strengthening teachers' Knowledge of Curriculum is also critical to ensure coherent sequencing of fraction concepts across grade levels and to prevent premature introduction of formal procedures. For future research, expanding the number of participants and incorporating classroom observations would provide a more comprehensive understanding of how PCK is enacted in authentic instructional settings. Additionally, comparative studies across different cultural and curricular contexts may further illuminate how teachers' PCK in fractions develops and how professional learning opportunities can effectively support teachers in progressing beyond algorithm-focused practices.

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