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Modified Premiun Reserve Analysis Using Commissioners Method in Whole Life Insurance

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Abstract

Premium reserve is the difference between the premium amount and the amount of compensation. The amount of whole life insurance premium reserve is calculated using the Commissioners method with the Woolhouse formula. The Commissioners reserve states the relationship between the net premium and the modified premium, namely between the modified β and modified α on any policy and the age at issue. The amount of the Commissioners reserve is influenced by the initial annuity value with three payments per year, the single premium of whole life insurance with three payments per year, and the amount of the modified premium with three payments per year. The initial annuity value of whole life insurance with the Woolhouse formula is influenced by the interest rate, the insured's survival rate, accelerated mortality, and the number of payments per year. This study uses a quantitative approach. This method is used to minimize the costs that are quite high in the first year. The research sample was taken using a purposive sampling technique, namely the Indonesian Mortality Table (female) in 2019. The simulation of the calculation of Commissioners' reserves was carried out with different nominal compound interest rates, the age of the insured was 35 years, the premium payment period for the annuity calculation was 30 years, and the premium payment period was 35 years**.** The amount of Commissioners' reserves for a 40-year-old female insured, with $i^{(3)} = 5{,}8838\%$ is Rp. 1,499,594.44. The amount of Commissioners' reserves for a 40-year-old female insured, with $i^{(3)} = 6{,}0377\%$ is Rp. 1,609,498.67. Also, the amount of Commissioners' reserves for a 40-year-old female insured, with $i^{(3)} = 6{,}0377\%$, is Rp. 1,670,281.62. The higher the nominal compound interest applied, the greater the Commissioners' reserves for a 40-year-old female insured. This research focuses on insurers who are bound by a life insurance contract to calculate the amount of modified premium reserves with m payments in one year using the Woolhouse formula in the Commissioners Method.

Keywords: Commissioners Method, Modified Premium, Modified Premium Reserve

1. Introduction

Life insurance is a type of insurance product between the insured and the insurer, where the insurer will pay benefits to the insured's family if the insured has an accident, is disabled, or dies. The amount of the annual premium paid by the insurer will affect the amount of benefits that will be received by his family. In an insurance contract, the insurance company is said to be the insurer, while the policyholder is said to be the insured (Khairunnisa et al., 2016). There are four types of life insurance based on the length of the policy coverage period, namely whole life insurance, n −years term insurance, pure endowment insurance, and endowment insurance (Artika et al., 2018).

Premium reserves are the difference between the amount of premium and the amount of compensation. Prospective reserves are reserves for a certain period of time, where the value of future compensation is reduced by the expected value of future premiums. The research article by Siska Fitriyani et al., 2021, states that the Canadian method is a modification of prospective reserves for dual-purpose life insurance which is more effective when compared to the Comissioners method. The premium reserve value obtained is greater when compared to the Comissioners method so that this value will benefit the insured. The research article by Rohaeni, 2007, states that the modification of dual-purpose life insurance reserves using the Zilmer method is a method that can be used to overcome insurance company losses because the calculation of the first year's net premium reserve is smaller when compared to the following year's net premium reserve. The research article by (Putri, 2020) states that the value of term life insurance premium reserves using the Commissioner method and the Woolhouse formula will experience a very large decrease when premium payments are made until the term follows the length of the insurance.

Based on previous research, the study conducted an update by analyzing the modification of premium reserves in whole life insurance. In a whole life insurance contract, the insurer will be bound by an agreement with the insured for the rest of his life. In this study, the insured makes premium payments three times a year. The premium paid by the insured will be saved as an official reserve that will be used to pay for insurance when a claim occurs. In the first year, the amount of premium reserves received by the insurer is not enough to cover the costs at the beginning of the year because the amount of reserves needed in the first year is greater when compared to the premiums received (Nur Indah Berliana Ratam, 2015). To overcome this, the premium reserve will be modified using the Commissioners method with the Woolhouse formula. The Woolhouse formula is one method for calculating payments made m times a year. This formula is a development of the Euler-MacLaurin formula. The Euler-MacLaurin formula is stated by (Studi Matematika, n.d.):

$$\int_0^\infty g(t)\,dt = h \cdot \sum_{k=0}^{\infty} g(kh) - \frac{h}{2} \cdot g(0) + \frac{h^2}{12} \cdot g'(0) - \frac{h^4}{720} \cdot g''(0) \pm \cdots \qquad (1)$$

The value of an annuity m times in one year is expressed by the equation:

$$\int_0^\infty g(t)\,dt = h \cdot \sum_{k=0}^{\infty} g(kh) - \frac{h}{2} \cdot g(0) + \frac{h^2}{12} \cdot g'(0) \qquad (2)$$

Suppose $g(t)$ is a function of the cash value of premium payments made once a year.

$$g(t) = v^t \cdot {}_tp_x \qquad (3)$$

where:

v^t is the present value of payments over t years

${}_tp_x$ is the probability of a person aged x years living to age $x + t$ years

The first derivative of the function $g(t)$ is

$$g'(t) = \frac{d}{dt}(v^t) \cdot \ {}_tp_x + v^t \cdot \frac{d}{dt}(\ {}_tp_x) = \frac{d}{dt}(e^{-\delta t}) \cdot \ {}_tp_x + v^t \cdot \frac{d}{dt}(\ {}_tp_x)$$

$$= -\delta \cdot e^{-\delta t} \cdot \ {}_tp_x - v^t \cdot \ {}_tp_x \cdot \mu_{x+t} \tag{4}$$

where:

$$\mu_{x+t} = -\frac{1}{2}\left(log(p_{x+t-1}) + log(p_{x+t})\right) \tag{5}$$

For $t = 0$, the first derivative of the function $g(t)$ is obtained as

$$g'(0) = -\delta \cdot e^{-\delta \cdot 0} \cdot \ {}_0p_x - v^0 \cdot \ {}_0p_x \cdot \mu_{x+0} = -(\delta + \mu_x) \tag{6}$$

As a result, the annuity value m times in one year and $h = 1$ is as follows (Laili et al., 2022):

$$\int_0^\infty g(t)\,dt = \sum_{k=0}^{\infty} g(k) - \frac{1}{2} \cdot g(0) + \frac{1}{12} \cdot [-(\delta + \mu_x)]$$

$$= \sum_{k=0}^{\infty} v^k \cdot \ {}_kp_x - \frac{1}{2} - \frac{1}{12} \cdot [\delta + \mu_x]$$

$$= \ddot{a}_x - \frac{1}{2} - \frac{1}{12} \cdot [-(\delta + \mu_x)] \tag{7}$$

Meanwhile, the annuity value of m times in one year and $h = \frac{1}{m}$ is as follows:

$$\int_0^\infty g(t)\,dt = \frac{1}{m} \cdot \sum_{k=0}^{\infty} g\left(\frac{k}{m}\right) - \frac{1}{2m} \cdot g(0) + \frac{1}{12m^2} \cdot g'(0)$$

$$= \frac{1}{m} \cdot \sum_{k=0}^{\infty} \left[v^{\frac{k}{m}} \cdot \ {}_{\frac{k}{m}}p_x\right] - \frac{1}{2m} - \frac{1}{12m^2} \cdot [\delta + \mu_x]$$

$$= \ddot{a}_x{}^{(m)} - \frac{1}{2m} - \frac{1}{12m^2} \cdot [\delta + \mu_x] \tag{8}$$

The value of the two annuities above has almost the same amount so it can be written as (Gerber, 1997) :

$$\ddot{a}_x{}^{(m)} - \frac{1}{2m} - \frac{1}{12m^2} \cdot [\delta + \mu_x] \approx \ddot{a}_x - \frac{1}{2} - \frac{1}{12} \cdot [-(\delta + \mu_x)]$$

$$\ddot{a}_x{}^{(m)} \approx \ddot{a}_x - \frac{1}{2} - \frac{1}{12} \cdot [-(\delta + \mu_x)] + \frac{1}{2m} + \frac{1}{12m^2} \cdot [\delta + \mu_x]$$

$$\approx \ddot{a}_x + \frac{1-m}{2m} + \frac{1-m^2}{12m^2} \cdot [\delta + \mu_x]$$

$$\approx \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2} \cdot [\delta + \mu_x]$$

$$\approx \frac{N_x}{D_x} - \frac{m-1}{2m} - \frac{m^2-1}{12m^2} \cdot [\delta + \mu_x] \tag{9}$$

The initial annuity value of whole life insurance for an insured aged x years using the Woolhouse formula is stated by:

$$\ddot{a}_x^{(m)} = \frac{N_x}{D_x} - \frac{m-1}{2m} \cdot \left(1 - v^h \cdot {}_h p_{x+t}\right) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_x - v^h \cdot {}_h p_x \cdot \left(\delta + \mu_{x+h}\right)\right) \tag{10}$$

The amount of the single premium for life insurance for an insured aged x years and with a coverage period of $n-t$ years using the Woolhouse Formula is

$$A_x^{(m)} = 1 - d^{(m)} \cdot \left(\frac{N_x}{D_x} - \frac{m-1}{2m} \cdot (1 - v^h \cdot {}_h p_{x+t}) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_x - v^h \cdot {}_h p_x \cdot (\delta + \mu_{x+h})\right)\right) \tag{11}$$

Also, the amount of the single premium for life insurance for an insured aged $x+t$ years and with a coverage period of $n-t$ years using the Woolhouse Formula is

$$A_{x+t}^{(m)} = 1 - d^{(m)} \cdot \left(\frac{N_{x+t}}{D_{x+t}} - \frac{m-1}{2m} \cdot (1 - v^h \cdot {}_h p_{x+t}) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_{x+t} - v^h \cdot {}_h p_{x+t} \cdot (\delta + \mu_{x+n})\right)\right) \tag{12}$$

The annual premium for whole life insurance with the Woolhouse formula for an insured aged x years and a premium payment period of h years is

$${}_h P_x^{(m)} = \frac{1 - d \cdot \left(\frac{N_x}{D_x}\right)}{\frac{N_x}{D_x} - \frac{m-1}{2m} \cdot (1 - v^h \cdot {}_h p_x) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_x - v^h \cdot {}_h p_x \cdot (\delta + \mu_{x+h})\right)} \tag{13}$$

The amount of the annual life insurance premium for an insured aged $x+t$ years is

$${}_h P_{x+t}^{(m)} = \frac{1 - d \cdot \left(\frac{N_{x+t}}{D_{x+t}}\right)}{\frac{N_{x+t}}{D_{x+t}} - \frac{m-1}{2m} \cdot (1 - v^h \cdot {}_h p_{x+t}) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_{x+t} - v^h \cdot {}_h p_{x+t} \cdot (\delta + \mu_{x+t+h})\right)} \tag{14}$$

One of the modification reserves is the Commissioners reserve. The commissioners reserve states the relationship between net premium and modification premium, namely between β modification and α modification on any policy and the age at issue (UNZILA NUR LAILI, 2022). This value is equal to the difference between the net premium of a whole life insurance with a premium payment period of 19 years issued at one year higher age and the net premium of a one-year term issued at an earlier age. The commissioners method is stated by the equation.

$$\beta^{com} - \alpha^{com} = {}_{19}P_{x+1} - c_x \tag{15}$$

where c_x is the natural premium or one-year term premium that is extended annually for a certain period of time. The natural premium is expressed by

$$c_x = v \cdot q_x \tag{16}$$

The Commissioners method with m payments per year can be expressed by

$$\beta^{com(m)} - \alpha^{com(m)} = {}_{19}P_{x+1}^{(m)} - c_x^{(m)} \tag{17}$$

$$\alpha^{com(m)} = \beta^{com(m)} - {}_{19}P_{x+1}^{(m)} + c_x^{(m)} \tag{18}$$

where $c_x^{(m)}$ is the natural premium m times the payoff expressed by

$$c_x^{(m)} = 1 - d^{(m)} \cdot \ddot{a}_{x:\overline{\overline{1}}|}^{(m)} - v \cdot p_x = 1 - d^{(m)} \cdot (1 + v \cdot p_x) - v \cdot p_x \tag{19}$$

and

$$d^{(m)} = m\left(1 - (1-d)^{\frac{1}{m}}\right) \tag{20}$$

Based on the Commissioners method with m payments, the modified net premium cash value can be expressed by

$$\alpha^{com(m)} + \beta^{com(m)} \cdot \left(\ddot{a}_{x:\overline{\overline{n}}|}^{(m)} - 1\right) = {}_hP_{x:\overline{\overline{n}}|}^{(m)} \cdot \ddot{a}_{x:\overline{\overline{n}}|}^{(m)} \tag{21}$$

Substitute the value of $\alpha^{com(m)}$ into the equation above to obtain

$$\beta^{com(m)} - {}_{19}P_{x+1}^{(m)} + c_x^{(m)} + \beta^{com(m)} \cdot \left(\ddot{a}_{x:\overline{\overline{n}}|}^{(m)} - 1\right) = {}_hP_{x:\overline{\overline{n}}|}^{(m)} \cdot \ddot{a}_{x:\overline{\overline{n}}|}^{(m)} \tag{22}$$

$$\beta^{com(m)} = {}_hP_{x:\overline{\overline{n}}|}^{(m)} + \frac{{}_{19}P_{x+1}^{(m)} - c_x^{(m)}}{\ddot{a}_{x:\overline{\overline{n}}|}^{(m)}} \tag{23}$$

Analogize equation (23) into the form of whole life insurance

$$\beta^{com(m)} = {}_hP_x^{(m)} + \frac{{}_{19}P_{x+1}^{(m)} - c_x^{(m)}}{\ddot{a}_x^{(m)}} \tag{24}$$

The amount of the Commissioners' reserves for whole life insurance for a person aged x years, with a premium payment period of h years, and m payments in one year, and t calculation periods for reserves and insurance money are stated as follows:

$$ {}_t^hV_x^{com(m)} = A_{x+t}^{(m)} - \beta^{com(m)} \cdot {}_h\ddot{a}_{x+t}^{(m)} \tag{25}$$

2. Research Method

This study uses a quantitative approach to the calculation of modified premiums and the calculation of modified premium reserves. The research team explored information from literature sources, books, references, and Indonesian mortality information (women) in 2019. The technique used in sampling is purposive sampling technique. This study uses data from the Indonesian Mortality Table IV 2019 (women). Several stages of research in calculating the amount of modified premium reserves are as follows:

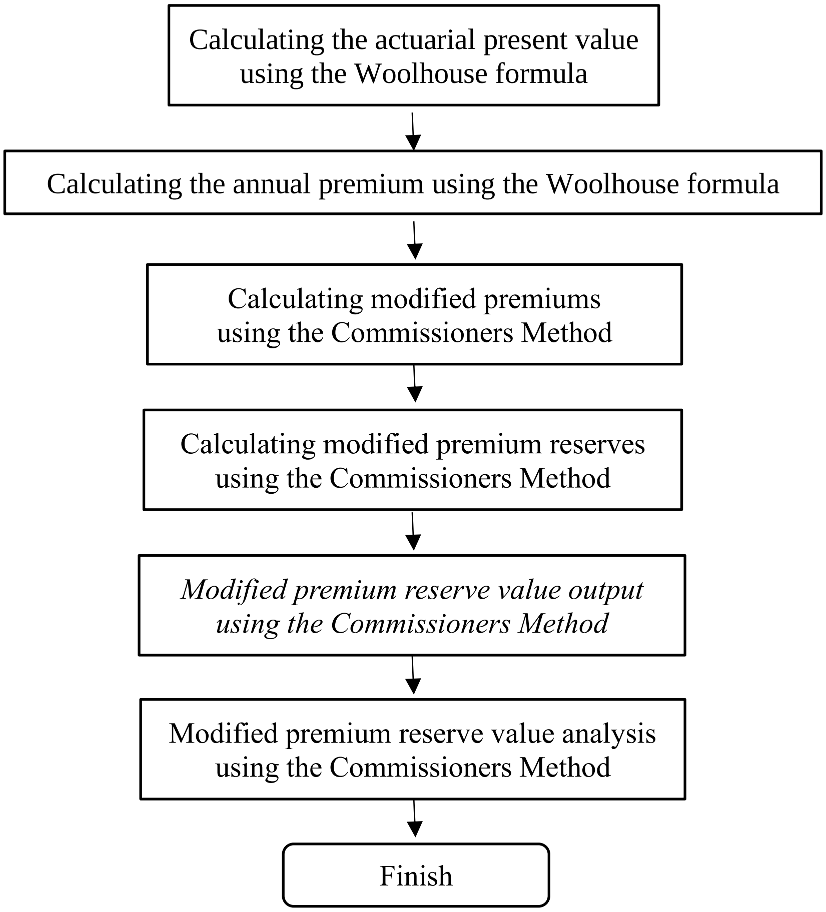

Figure 1. Research Steps

3. Results and Discussions

Given the age of the insured is 35 years, the premium payment period for the annuity calculation is 30 years, and the premium payment period is 35 years. And the amount of money market interest rate (IndoNIA) moving around the BI-rate is 6.16% (period October 15, 2024). As a result, the amount of nominal compound interest paid three times a year is $i^{(3)} = 6{,}0377\%$. With this research approach, there are several values that are calculated, including:

$$
\begin{aligned}
{}_{30}\ddot{a}_{35}^{(m)} &= \frac{N_{35}}{D_{35}} - \frac{m-1}{2m} \cdot \left(1 - v^{30} \cdot {}_{30}p_{35}\right) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_{35} - v^{30} \cdot {}_{30}p_{35} \cdot (\delta + \mu_{65})\right) \\
&= \frac{204.495{,}25}{12.668{,}86} - \frac{3-1}{2 \cdot 3} \cdot (1 - 0{,}172264 \cdot 0{,}897935) - \frac{3^2-1}{12 \cdot 3^2} \\
&\qquad \cdot \left(0{,}02546 + 0{,}00033454 - 0{,}172264 \cdot 0{,}897935 \cdot (0{,}02546 + 0{,}00301138)\right) \\
&= 15{,}8584262
\end{aligned}
$$

The initial annuity value of a 35-year-old person with a coverage period of 30 years is 15.8584262. The actuarial present value:

$$A_{35}^{(m)} = 1 - d^{(m)} \cdot \left(\frac{N_{35}}{D_{35}} - \frac{m-1}{2m} \cdot \left(1 - v^{30} \cdot {}_{30}p_{35}\right) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_{35} - v^{30} \cdot {}_{30}p_{35} \cdot (\delta + \mu_{65}) \right) \right)$$

$$= 1 - 0{,}058055 \cdot \left(\frac{204.495{,}25}{12.668{,}86} - \frac{3-1}{2 \cdot 3} \cdot (1 - 0{,}172264 \cdot 0{,}897935) - \frac{3^2-1}{12 \cdot 3^2} \cdot \left(0{,}02546 + 0{,}00033454 - 0{,}172264 \cdot 0{,}897935 \cdot (0{,}02546 + 0{,}00301138)\right) \right)$$

$$= 0{,}0780075183$$

If a benefit of IDR 100,000,000 is given, the single premium (actuarial present value) for life insurance for a 35-year-old person with a coverage period of 30 years is IDR 7,800,751.83.
The annual premium:

$${}_{30}P_{35}^{(m)} = \frac{1 - d \cdot \left(\frac{N_{35}}{D_{35}}\right)}{\frac{N_{35}}{D_{35}} - \frac{m-1}{2m} \cdot \left(1 - v^{30} \cdot {}_{30}p_{35}\right) - \frac{m^2-1}{12m^2} \cdot \left(\delta + \mu_{35} - v^{30} \cdot {}_{30}p_{35} \cdot \left(\delta + \mu_{65}\right) \right)}$$

$$= \frac{1 - (0{,}056939)\left(\frac{204.495{,}25}{12.668{,}86}\right)}{\frac{204.495{,}25}{12.668{,}86} - \frac{3-1}{2 \cdot 3} \cdot (1 - 0{,}172264 \cdot 0{,}897935) - \frac{3^2-1}{12 \cdot 3^2} \cdot (0{,}02546 + 0{,}00033454 - 0{,}172264 \cdot 0{,}897935 \cdot (0{,}02546 + 0{,}00301138))}$$

$$= \frac{1 - (0{,}056939)(16{,}14157)}{15{,}8584262}$$

$$= 0{,}0051019478$$

The modified premium value:

The present value with three payments per year is

$$v^{(3)} = \frac{1}{1 + i^{(3)}} = 0{,}943061$$

The amount of natural premium is

$$c_{35}^{(3)} = 1 - d^{(3)} \cdot \ddot{a}_{35:\overline{1}|}^{(3)} - v \cdot p_{35}$$

$$= 1 - (0{,}058055) \cdot (0{,}664754126) - (0{,}943061) \cdot (0{,}9992)$$

$$= 1 - 0{,}0385923 - 0{,}9423066$$

$$= 0{,}0191011$$

The nominal discount value is

$$d^{(3)} = 3 \cdot \left(1 - (1 - 0{,}056939)^{\frac{1}{3}}\right) = 0{,}058055$$

Based on the Commissioners method with three payments, the modified net premium cash value can be expressed by

$$\beta^{com(3)} = {}_{30}P_{35}^{(m)} + \frac{{}_{19}P_{36}^{(3)} - c_{35}^{(3)}}{{}_{30}\ddot{a}_{35}^{(m)}} = 0{,}0051019478 + \frac{0{,}005371 - 0{,}019101}{15{,}8584262}$$
$$= 0{,}004232141$$

If given a benefit of IDR 100,000,000,-, the amount of the modified premium for whole life insurance for a person aged 35 years with a coverage period of 30 years is IDR 423,214.1,-.

The premium reserve value:

$$ {}_{1}^{30}V_{35}^{com(3)} = A_{36}^{(3)} - \beta^{com(3)} \cdot {}_{30}\ddot{a}_{36}^{(3)}$$
$$= 0{,}082265862 - (0{,}004232141) \cdot (15{,}7852793)$$
$$= 0{,}0154603283$$

If given a benefit of IDR 100,000,000,-, the amount of the modified premium reserve for whole life insurance for a person aged 35 years with a coverage period of 30 years is IDR 1.546.032,83,.

4. Conclusions

Premium reserve is the difference between the premium amount and the amount of compensation. Whole life insurance premium reserve will be calculated to help the insurance company to reserve its money in case of an undesirable event during the insurance contract. The amount of whole life insurance premium reserve is calculated using the Commissioners method with the Woolhouse formula. The Commissioners reserve states the relationship between the net premium and the modified premium, namely between the modified β and modified α on any policy and the age when issued. The amount of the Commissioners reserve is influenced by the initial annuity value with three payments per year, the single premium of whole life insurance with three payments per year, and the amount of the modified premium with three payments per year. The initial annuity value of whole life insurance with the Woolhouse formula is influenced by the interest rate, the insured's chances of survival, accelerated mortality, and the number of payments per year. The higher the nominal compound interest applied, the greater the Commissioners reserve set for the insured. Also, the older a person is, the more expensive the premium that must be paid. One reason is that customers with older ages generally have higher health risks. As a result, insurance companies will set higher premium reserves for older customers as funds reserved by the company in case the customer files a claim.

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